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The motion of particles near a bubble in a gas-fluidized bed

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Most models of gas bubbles in fluidized beds are based on the assumption of an empty central region, the void, surrounded by a 'cloud' or 'shell' of particles whose voidage is larger than that of the remote emulsion phase. Batchelor & Nitsche (1994) investigated the formation of a void by tracking the paths of particles initially within a buoyant 'blob' of gas that has the form of a toroidal vortex. They showed that the particles dropped through the floor of the blob under the influence of gravity, leaving it empty. This paper extends their method to particles initially outside the blob. It is shown that inertia allows these particles to penetrate the blob and it is the extent of this penetration that determines the size of the void. The void is nearly as large as the blob for small, light particles, but becomes smaller relative to the blob with increasing particle size and weight until it disappears altogether. This provides an explanation for experimental observations of voids smaller than the blob (or 'cloud' as it is sometimes known), and suggests that when examining bubbles in a gas-fluidized bed the most significant dimension is the diameter of the blob and not that of the void.

1. Introduction

The motion of particles and gas near bubbles in a fluidized bed suffers from the same fundamental problem as the modelling of most multi-phase phenomena: it involves the motion of two components – particles and gas – whose interface is unknown and changing and whose interaction is known only crudely. To make problems tractable assumptions about the nature of the particulate phase have to be made. The most common approach is to use the two-fluid model where the particles are treated as a fluid superimposed on the gas phase (i.e. the two phases are interpenetrating). This allows the behaviour of the particles to be described using conservation equations which include terms that express the interaction between the fluid and the particles. In other words the individual nature of the particles is removed, apart from their overall mass, momentum and energy exchanges with the surrounding gas, and the two phases act like compressible fluids with effective densities equal to the proportion of the volume of the mixture they occupy (often denoted using voidage ϵ) multiplied by their true densities.

The two-fluid approach was originally used for the modelling of bubbles in a fluidized bed by Davidson (1961) in a steady model where the bubble void was represented by an empty sphere embedded in a uniform emulsion. The particles and gas were both modelled as inviscid fluids with a common pressure field and the motion near the bubble was found by potential flow theory. No account was taken of the interaction between particles. The model predicted the existence of a vortex of gas around the void (often called 'the cloud') which is separate from the gas in the rest of the bed and which moves up through it with the void (Davidson 1961). This was confirmed experimentally for two-dimensional beds by Rowe, Partridge & Lyall (1964) and for three-dimensional liquid beds by Davidson (1961). Attempts were made to extend the model by including a particle momentum equation (Jackson 1963; Murray 1963), but the approximations necessary for this meant that very little was added to the Davidson model (Jackson 1971; Collins 1989). Pyle & Rose (1965) modified Davidson's model by deriving an equation for the stream function of the vortex that satisfied certain imposed boundary conditions at the edge of the void. Davidson's model has also been extended to take into account some forms of the voidage variation seen in the bed outside the void, with some interesting results (Collins 1989; Benveniste, Kinrys & Qassim 1983). Another modification was made by Buyevich *et al.* (1995) following the experimental work of Yates, Cheesman & Sergeev (1994) where the emulsion phase of the bed was considered to be analogous to a dense gas composed of hard spheres in which the random concentration fluctuations of the dense phase were found by calculating its 'temperature'.

A different approach was taken by Batchelor (1974) where it was shown that for small particle and bubble Reynolds numbers the equations of motion of the gas and the emulsion were analogous to those for a small bubble in a viscous fluid. These could be solved analytically whereupon it was found that if there was slip between the particles and fluid then a cloud would form around the void.

2. Particle tracking

2.1. *The model of Batchelor & Nitsche*

The two-fluid model has produced important information concerning the motion near bubbles; however the existence and size of the void has always been assumed. Further insights can be gained by following a complementary approach where the individual nature of particles is restored and the tracks of representative particles are followed, i.e. a gas flow field is specified and then the path that an individual particle would have within it is calculated. As in the Davidson model the flow field of one of the phases is assumed, but the particles are not assumed to act like a fluid. Batchelor has shown in two papers (Batchelor 1988, 1993) how buoyant regions or 'blobs' of higher voidage might form in a fluidized bed through hydrodynamic instability. In a further paper (Batchelor & Nitsche 1994) particle tracking was used to show how voids might form from these blobs. The buoyant blob of gas was expected to be a vortex but its form was unknown so the convenient Hill's spherical vortex was assumed. This has a stream function

$$\psi = \tfrac{1}{2}u_0 r^2 \sin^2\theta \left(1 - \left(\frac{r}{R}\right)^2\right) \tag{2.1}$$

where a spherical coordinate system (r, θ) is used, R is the radius of the vortex, and u_0 is the velocity representative of the fluid motion within the bubble; u_0 was assumed to have the same order of magnitude as the rise velocity W of the bubble relative to the surrounding emulsion. As in Batchelor & Nitsche (1994), it will be assumed for simplicity that

$$u_0 \approx W \approx K(gR)^{1/2} \tag{2.2}$$

and K is equal to 1.0.

The path of a particle was tracked by placing it initially on the equator of the vortex and then solving its equation of motion:

$$\frac{d\boldsymbol{v}}{dt} = \boldsymbol{g} - \frac{\boldsymbol{v} - \boldsymbol{u}}{\tau_0} D_{\boldsymbol{v}-\boldsymbol{u}} \tag{2.3}$$

where $\boldsymbol{v}$ is the particle velocity; $\boldsymbol{u}$ is the gas velocity; τ_0 is the particle viscous relaxation time given by

$$\tau_0 = \frac{2d_p^2 \rho_p}{9\mu}, \tag{2.4}$$

where d_p is the particle diameter, ρ_p is the particle density, and μ is the gas viscosity; and $D_{\boldsymbol{v}-\boldsymbol{u}}$ is the drag between the particle and the fluid given by an empirical correlation:

$$D_{\boldsymbol{v}-\boldsymbol{u}} = \left(1 + 0.13\left(\frac{2d_p|\boldsymbol{v} - \boldsymbol{u}|\rho_f}{\mu}\right)\right)^2. \tag{2.5}$$

It was assumed that the blob was sufficiently empty for there to be negligible interaction between particles (as in the Davidson model) and for the influence of the particles to be sufficiently small for their motion not to affect that of the vortex. It was emphasized that the purpose of the model was not to represent the flows accurately, but to show that a void can form through the rapid expulsion of particles from a blob when the interior gas flow is toroidal with a velocity of a certain magnitude.

The model showed that particles within a blob drop through its bottom under the influence of gravity within a short timescale. Small particles might make several circuits of the bubble as they are swept round it by the gas vortex before being expelled, but the number and the size of the circuits decrease with increasing particle diameter until all the particles fall immediately, and nearly straight, out of the blob.

In Batchelor & Nitsche (1994) it was expected that the entire blob would be evacuated and surrounded by a thin cloud where the voidage lay between that of the emulsion and a value of one. Though the model provides an explanation for how particles inside a blob can be expelled, it takes no account of particles with initial positions anywhere other than on the blob's equator; in particular no reference is made to particles initially outside the blob. The model does not provide any explanation for the existence of the shell of increased voidage surrounding a void or provide a mechanism for the support of the roof of the void. As before the existence of a spherical void was assumed to be inevitable; the purpose of the model was to show how it might be vacated.

2.2. *Extension of the model*

The calculations of Batchelor & Nitsche (1994) demonstrated the importance of the inertia and weight of particles for their motion: it is these forces that cause them to slip relative to the fluid flow and be expelled from the blob; however, whether inertia might allow particles to enter a blob from outside was not considered. An alternative view of the structure of the blob is that there is a central void from which these particles are excluded surrounded by a cloud that lies between the edge of the void and the edge of the blob which initially exterior particles penetrate. To test whether this might be credible, the model described in Batchelor & Nitsche (1994) has been extended to particles initially outside the blob.

The gas flow around the blob was assumed to be potential flow around a sphere

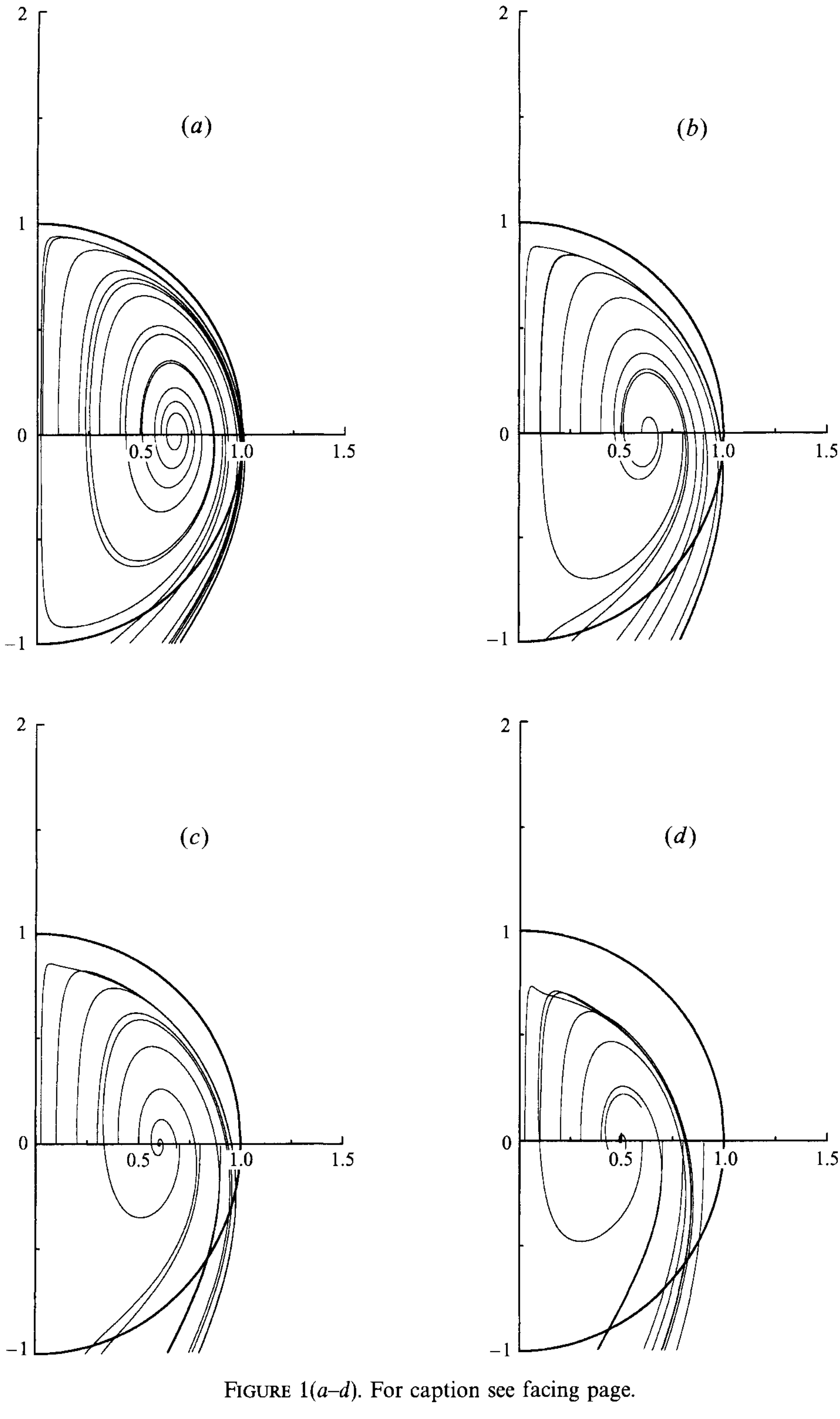

FIGURE 1(*a–d*). For caption see facing page.

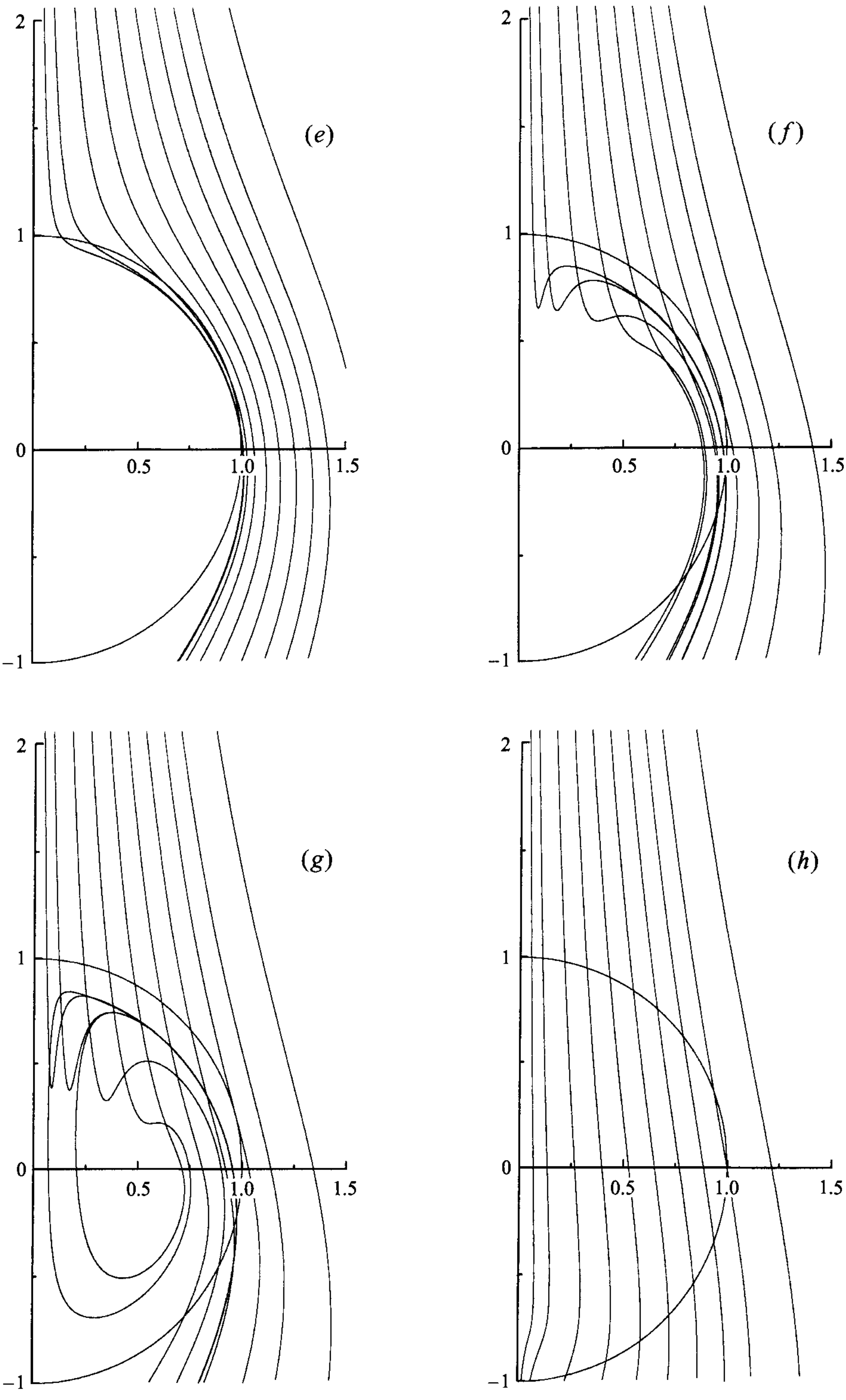

FIGURE 1. Trajectories for particles originally inside (*a–d*) and outside (*e–h*) a bubble in a gas-fluidized bed. (*a,e*) $d_p = 40$ μm; (*b,f*) 60 μm; (*c,g*) 70 μm; (*d,h*) 100 μm $\rho_p = 1000$ kg m^{-3}. The coordinates have been normalized with respect to the radius of the blob.

with the stream function

$$\psi = \tfrac{1}{2} W r^2 \sin^2\theta \left(1 - \left(\frac{R}{r}\right)^3\right). \tag{2.6}$$

The use of this equation in conjunction with Hill's vortex (2.1) means that the tangential velocity component of the fluid at the boundary of the blob is discontinuous. Discontinuous boundary conditions have been admitted in past models: for example Pyle & Rose (1965) had a continuous tangential fluid velocity distribution over the void boundary, but the total velocity distribution was discontinuous; Batchelor (1974) ensured that there was continuity from the emulsion to the void, but this entailed a discontinuous velocity distribution for the fluid phase. On balance the best boundary condition would probably be for the distributions of the components of the fluid velocity to be continuous over the blob boundary. This might be achieved by modifying the equation of the vortex using the procedure followed by Pyle & Rose (1965) where the flow within the vortex is derived from a general stream function for an axisymmetric object. This results in an equation with the same form as (2.1) but with the 1/2 replaced by 3/4. As the modification only results in a strengthening of the vortex, and in the light of the assumption that u_0 is equal to W when this may be only true to an order of magnitude, it was not adopted so that continuity with the results of Batchelor & Nitsche (1994) could be maintained.

Particles were started 5 radii above the centre of the blob. As in Batchelor & Nitsche (1994) the initial particle velocity at the point of release was chosen so that the initial particle acceleration was zero, and the same gas and particle properties were used. As in that instance the equations were non-dimensionalized and solved numerically using a fourth-order Runge–Kutta integration for motion confined to a plane containing the bubble axis.

3. Results

The results are shown on the right-hand side of figure 1 with corresponding diagrams of the motion of particles starting inside the bubble on the left-hand side. Note that distances are normalized with respect to R. The particle trajectories in the lower portion of the bubble are likely to be unrealistic owing to the influence of the wake that follows real bubbles, and the recirculation of particles will then be disrupted. Small particles follow the motion of the gas closely so that very little penetration of the blob takes place and, once particles have been expelled from it, the blob remains nearly empty. However as the particle diameter increases, the blob is increasingly penetrated by particles from outside. These are decelerated until their motion is dominated by that of the vortex and they follow paths similar to those of particles that originated in the blob. There is a continuous supply of particles from outside the blob so the areas they pass through will be perpetually occupied. If the particle diameter is increased further the penetration by the particles is greater so that the size of the void relative to that of the blob becomes smaller until eventually they pass straight through the blob with little deviation and no void would be in evidence. Figure 2 shows the effect on the paths of 60μm diameter particles of increasing their density by one half. As might be expected, owing to the larger inertia of the particles greater penetration of the blob takes place and the void size is reduced. The size of the void for particles of a given size and density also depends on the strength of

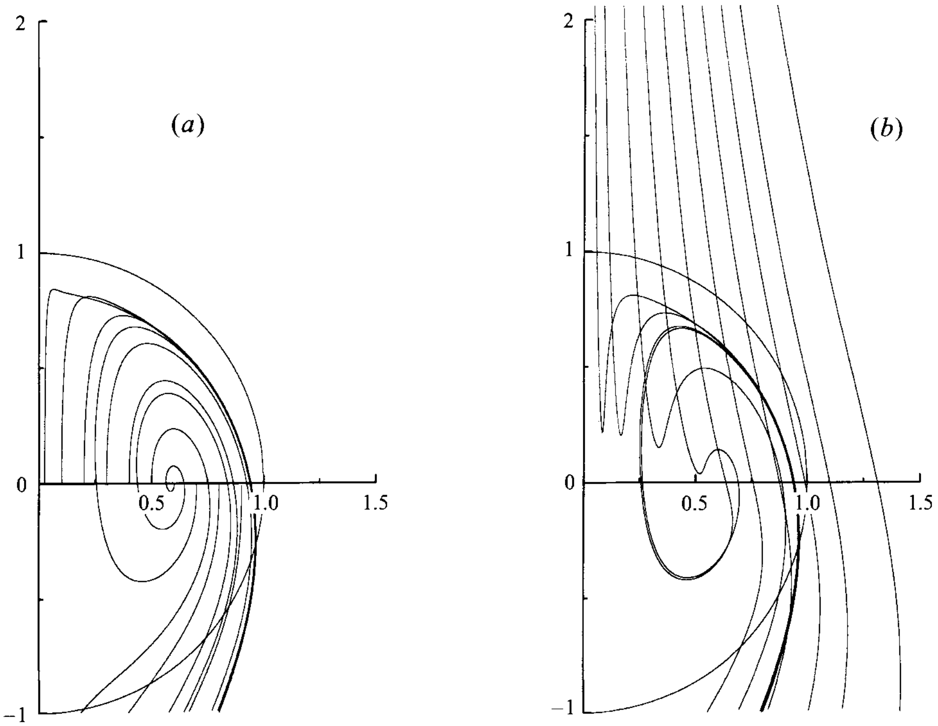

FIGURE 2. Trajectories for particles originally inside and outside a bubble in a gas-fluidized bed where $d_p = 60$ μm and ρ_p has been increased from 1000 to 1500 kg m^{-3}.

the vortex which in both Batchelor & Nitsche (1994) and in this paper is an order of magnitude estimate.

This simple model of particle paths close to bubbles demonstrates how voids may form and how the roof of a void might be supported in a particulate flow where, unlike in bubbly flows, the stabilizing agent of surface tension is not present. In addition it shows how the cloud might extend beyond a void into the surrounding emulsion. Batchelor (1974) proposed that the existence of the cloud was the result of slip between particles and gas; however the particles and fluid outside the void were treated together as a liquid and the boundary of the void was only preserved through the selection of appropriate boundary conditions. It is not possible to identify the edge of the shell where voidage exceeds that of the emulsion with that of the cloud because the gas flow, and hence the particle distribution, will be distorted owing to flow around the blob.

3.1. *Liquid-fluidized beds*

It is possible to extend the model to the case of liquid-fluidized beds though the degree of idealization is even greater than for gas fluidization. Batchelor & Nitsche (1994) adapted the equation of motion of a particle, (2.3), to include the effects of virtual mass. It was found that the particles followed the liquid flow much more closely than in a gas-fluidized bed, and made several circuits of the bubble before being expelled. The result of this was that the blobs modelled in a liquid-fluidized bed took a great deal longer to empty of particles than those in a gas-fluidized bed: this is possibly why bubbles are rarely seen in liquid-fluidized beds. Using the same equations we have calculated the paths of particles that start outside the bubble, but for the range of parameters investigated in Batchelor & Nitsche (1994) the particles were merely swept around the outside of the blob and very little penetration took place, as might be expected.

4. Discussion

4.1. *Limitations of the model*

The assumptions made for the model do not allow quantitative predictions: the behaviour of real particles of a given diameter is likely to be different to that shown in figure 1. Possible differences between the model and a real bubble include:

the form and strength of the vortex;

the effect of the bubble's wake on the particle trajectories towards the bottom of the bubble;

significant interaction between particles;

the effect of the motion of particles on the fluid flow.

However, as in Batchelor & Nitsche (1994), the model is not intended to yield accurate, quantitative results, but to show qualitatively how bubbles might form and develop; on these terms the model contains all the necessary essential features. A stable, discrete blob of gas has been shown to form around a void and move with it, and, in common with gas bubbles in a liquid, probably has the form of a toroidal vortex. Calculations performed by Batchelor & Nitsche (1994) indicated that over some ranges of particle diameters there was a possibility that the particles may significantly affect the motion of the gas unless their concentration was dilute; however, in experiments bubbles appear to hold a shape roughly like that of a spherical cap and so it might be inferred that though the details of the gas flow around them may change, their general form – that of a discrete vortex – remains the same while they are in existence. In addition the motion of a particle will be dominated by its weight, its inertia, and drag; it is difficult to envisage any other significant forces operating in a fluidized bed over a wide range of particle Reynolds numbers. The model should be a good general guide to behaviour in bubbles in a fluidized bed, but it cannot predict realistic particle trajectories for a given set of parameters; while the changes in behaviour shown in figure 1 can be expected, the particle parameters at which they take place cannot be confidently predicted.

4.2. *Significance of the results*

A consequence of this line of approach to bubbles in a fluidized bed is the minor importance that the void has when considering the overall mechanics of the bed. Historically bubbles have been considered as analogous to gas bubbles in liquids with the edge of the void identified with the edge of the bubble. The approach described above suggests that the void is an artefact of the behaviour of a blob, not a cause of it, and voids in a fluidized bed are very different in origin and structure to those found in liquids. The critical dimension that determines the motion of a bubble for a given powder is the diameter of the blob, which may be very different from that of the void. A possible example of when this might be important is when calculating the rise velocity of a bubble in a fluidized bed. This is often described by using the equation of Davies & Taylor (1950), (2.2) (though usually with a different value for K), developed for the velocity of rise of gas bubbles in liquid by approximating the gas flow around the front of a bubble as the flow of a frictionless fluid around a sphere. The equation has often been directly applied to bubbles in gas-fluidized beds by replacing the bubble radius with the void radius;

however in a gas-fluidized bed the external flow is around the cloud not the void, and the cloud radius should be used in the equation, not the void radius. This would not matter if the void radius is a fixed fraction of cloud radius; but if it varies with circumstances, as indicated by the above theory, this may account for the large degree of scatter seen if bubble velocity is plotted against void diameter, as shown in Davidson , Harrison & Guedes de Carvalho (1977) for example, and why in some studies, such as that of Rowe & Partridge (1965), values of K are found that are consistantly in excess of that calculated by Davies & Taylor (1950).

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REFERENCES

BATCHELOR, G. K. 1974 Low-Reynolds-number bubbles in fluidised particles. *Arch. Mech.* **26**, 339–353.

BATCHELOR, G. K. 1988 A new theory of the instability of a uniform fluidized bed. *J. Fluid Mech.* **193**, 75–110.

BATCHELOR, G. K. 1993 Secondary instability of a gas-fluidized bed. *J. Fluid Mech.* **257**, 359–371.

BATCHELOR, G. K. & NITSCHE, J. M. 1994 Expulsion of particles from a buoyant blob in a fluidized bed. *J. Fluid Mech.* **278**, 63–81.

BENVENISTE, D. E., KINRYS, S. & QASSIM, R. Y. 1983 Flow past the fluidization bubble: Effect of voidage variation. In *Fluidization* (ed. D. Kunii & R. Toei), pp. 9–14, Engineering Foundation, New York.

BUYEVICH, Y. A., YATES, J. G., CHEESMAN, D. & WU, K. T. 1995 A model for the distribution of voidage around bubbles in a fluidized bed. *Chem. Engng Sci.* **50**, 3155–3162.

COLLINS, R. 1989 A model for the effects of the voidage distribution around a fluidization bubble. *Chem. Engng Sci.* **44**, 1481–1487.

DAVIDSON, J. F. 1961 Symposium on fluidisation – Discussion. *Trans Inst. Chem. Engrs* **39**, 232.

DAVIDSON, J. F., HARRISON, D. & GUEDES DE CARVALHO, J. R. F. 1977 On the liquidlike behaviour of fluidized beds. *Ann. Rev. Fluid Mech.* **9**, 55–86.

DAVIES, R. M. & TAYLOR, G. I. 1950 The mechanics of large bubbles rising through extended liquids and through liquids in tubes. *Proc. R. Soc. Lond.* A **200**, 375–390.

JACKSON, R. 1963 The mechanics of fluidized beds part II: The motion of fully developed bubbles. *Trans. Inst. Chem. Engrs* **41**, 22–28.

JACKSON, R. 1971 Fluid mechanical theory. In *Fluidization* (ed. J. F. Davidson, & D. Harrison), chap. 3. Academic Press.

MURRAY, J. D. 1963 On the mathematics of fluidization. Part 2. Steady motion of fully developed bubbles. *J. Fluid Mech.* **22**, 57–80.

PYLE, D. L. & ROSE, P. L. 1965 Chemical reaction in bubbling fluidized beds. *Chem. Engng Sci.* **20**, 25–31.

ROWE, P. N. & PARTRIDGE, B. A. 1965 An X-ray study of bubbles in fluidised beds. *Trans Inst. Chem. Engrs* **43**, 157–175.

ROWE, P. N., PARTRIDGE, B. A. & LYALL, E. 1964 Cloud formation around bubbles in gas-fluidized beds. *Chem. Engng Sci.* **19**, 973–985.

YATES, J. G., CHEESMAN, D. J. & SERGEEV, Y. A. 1994 Experimental observations of voidage distribution around bubbles in a fluidized bed. *Chem. Engng Sci.* **49**, 1885–1895.